Many-Flavor Domain Wall Fermions and Fixed Topology

Meifeng Lin

Yale

New Horizons for Lattice Computations with Chiral Fermions
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Outline

Motivation

- SU(N) gauge theories with many flavors of fermions may exhibit behaviors that are fundamentally different from QCD.
- Increasing N_f causes the gauge coupling to run slowly, and "walking" may occur, which could lead to some properties desirable for Technicolor model building.
- Specifically, we are interested in
  - enhanced chiral condensate
  - small S parameter
  - new pattern in hadron spectrum
- When $N_f > N_f^c$, the theory becomes conformal, and spectrum vanishes at the chiral limit. Also on our wishlist is to
  - locate N_f^c non-perturbatively

The Lattice Strong Dynamics Collaboration

Tom Appelquist	Yale U.
Ron Babich	NVIDIA
Rich Brower	Boston U.
Michael Buchoff	LLNL
Michael Cheng	Boston U.
Mike Clark	NVIDIA
Saul Cohen	U. Washington
George Fleming	Yale U.
Joe Kiskis	U. of Carlifornia, Davis
Meifeng Lin	Yale U.
Heechang Na	ANL
Ethan Neil	Fermilab
James Osborn	ANL
Claudio Rebbi	Boston U.
David Schaich	Boston U.
Chris Schroeder	LLNL
Sergey Syritsyn	LBL
Joe Wasem	LLNL
Oliver Witzel	Boston U.
Gennady Voronov	Yale U.
Pavlos Vranas	LLNL

Gauge Ensembles

- Fermion action: domain wall fermions.
- Gauge action: Iwasaki.
- Ensemble size: Typically ~ 1200 MD trajectories.

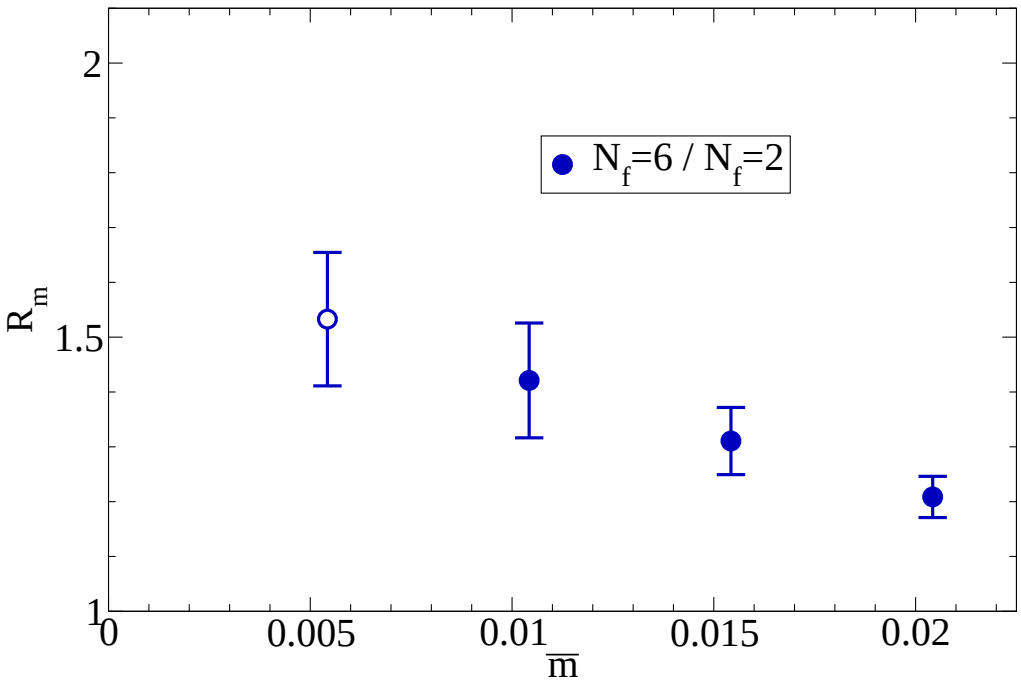
[Cost grows as $N_f^{3/2}$]

N_f	Volume	L_s	β	am_f	am_{res}
2	$32^3 \times 64$	16	2.70	0.005...0.030	2.5×10^{-5}
6	$32^3 \times 64$	16	2.10	0.005...0.030	8.2×10^{-4}
10	$32^3 \times 64$	16	1.95	0.005...0.030	1.7×10^{-3}

- For $N_f = 2$, $aM_V \approx 0.21 \Rightarrow a^{-1} \approx 3.7$ GeV.
- For $N_f = 6$ and 10, match aM_V to within 10% of $N_f = 2$.

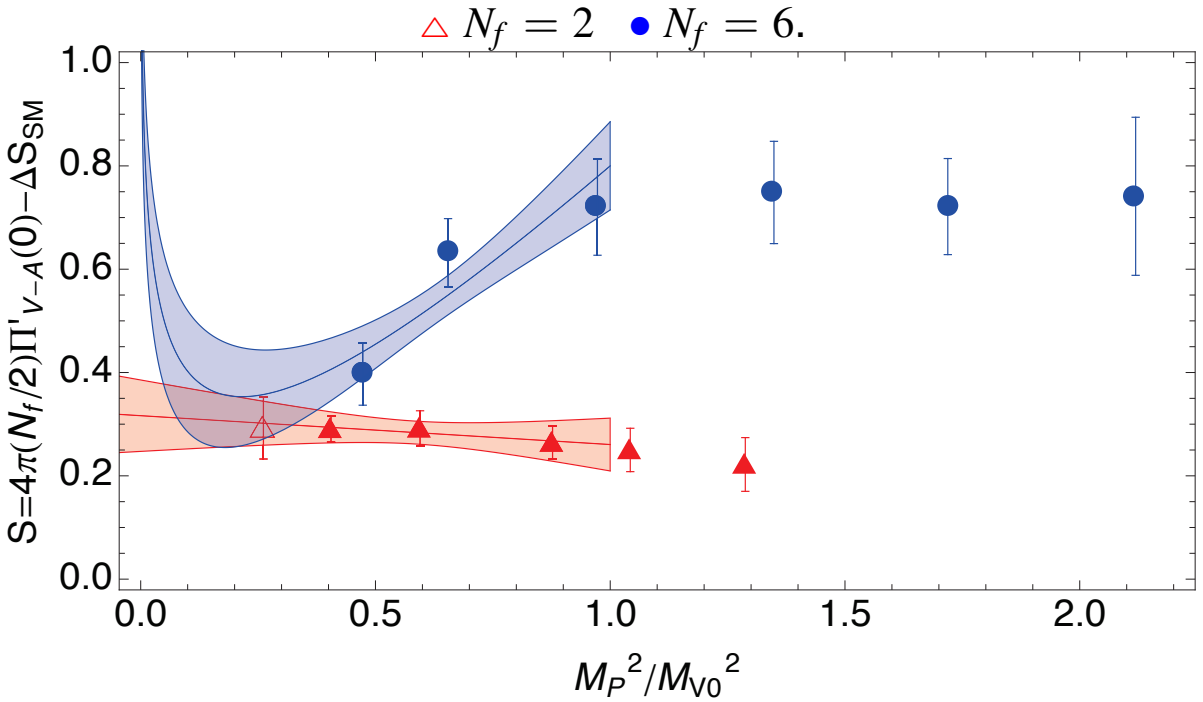
Condensate Enhancement

$$R_m \equiv \frac{M_m^2}{2mF_\pi}_{(N_f=6)} \Big/ \frac{M_m^2}{2mF_\pi}_{(N_f=2)}$$



T. Appelquist *et al.* (LSD Collaboration), PRL104:071601,2010

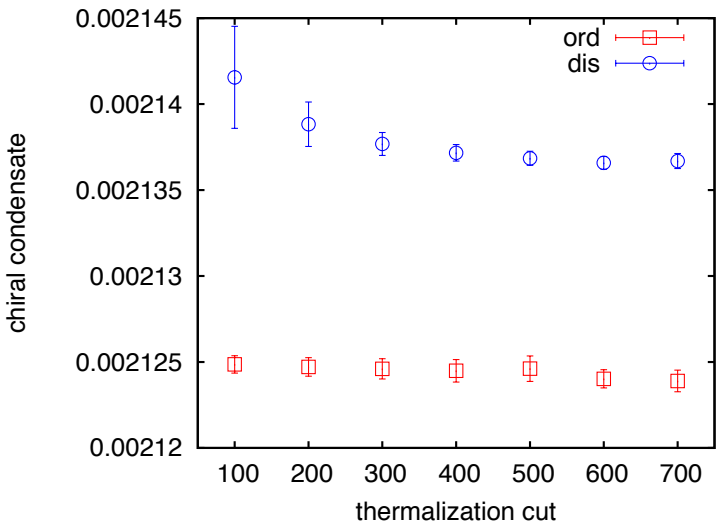
The S Parameter

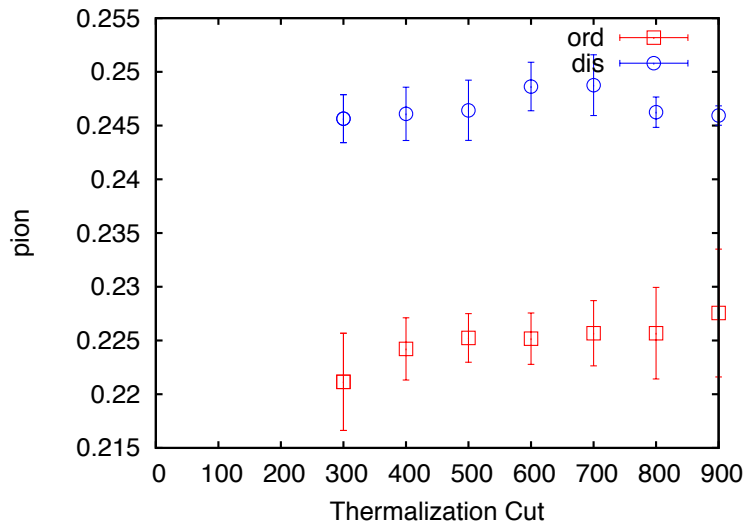


T. Appelquist *et al.* (LSD Collaboration), PRL 106 (2011) 231601

A New Observation for $N_f = 10$

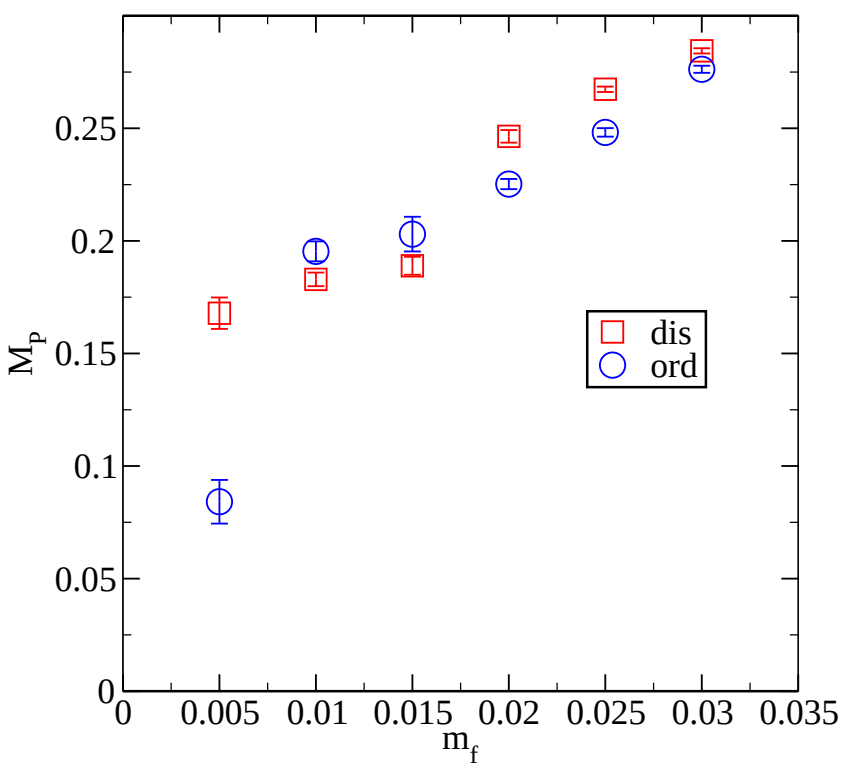
- Two independent ensembles for each quark mass, with ordered and disordered start.
- The ensembles thermalize to different values
- NOT observed for $N_f = 2$ or 6.

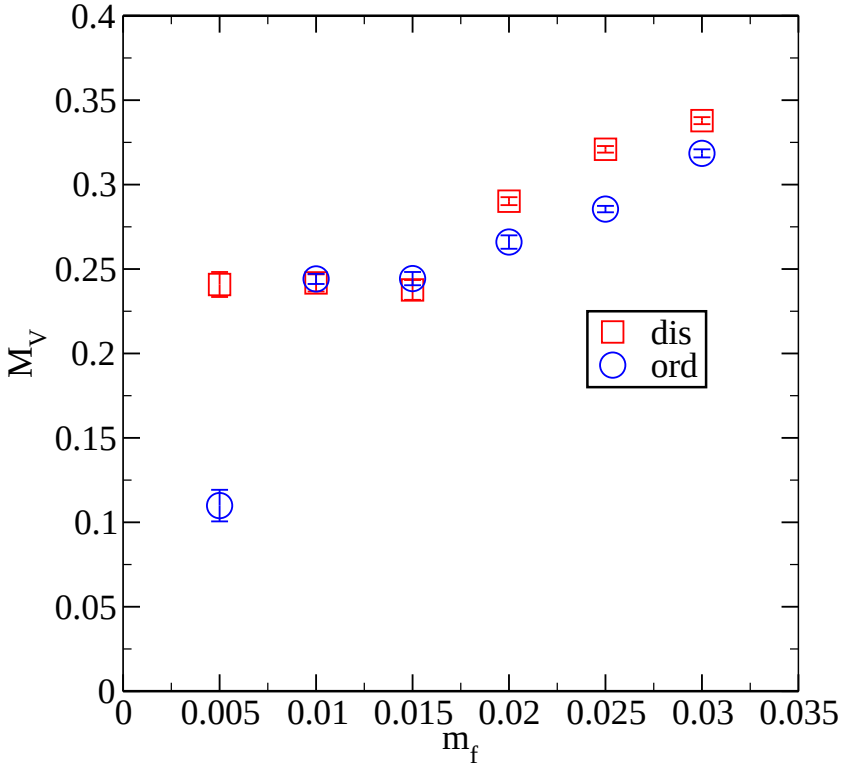




Starting-Config. Dependence

Statistically significant diffference between the ordered-start and disordered-start ensembles are observed for almost all the ensembles.





Slowly-moving/Fixed Topology

m_f	start	trajectory range	Q
0.005	O	500 - 1319	0
0.005	D	500 - 919	3
0.010	O	500 - 1373	0
0.010	D	500 - 1259	1
0.015	O	500 - 1204	0
0.015	D	500 - 1450	-1
0.020	O	500 - 1239	0
0.020	D	500 - 1221	12
0.025	O	500 - 1256	0
0.025	D	500 - 609	-19
		610 - 942	-18
		943 - 1095	-17
		1096-1097	-16
		1098 - 1382	-17
		1383 - 1484	-16
0.030	O	500 - 1415	0
0.030	D	500 - 1227	9

Effects of Fixed Topology

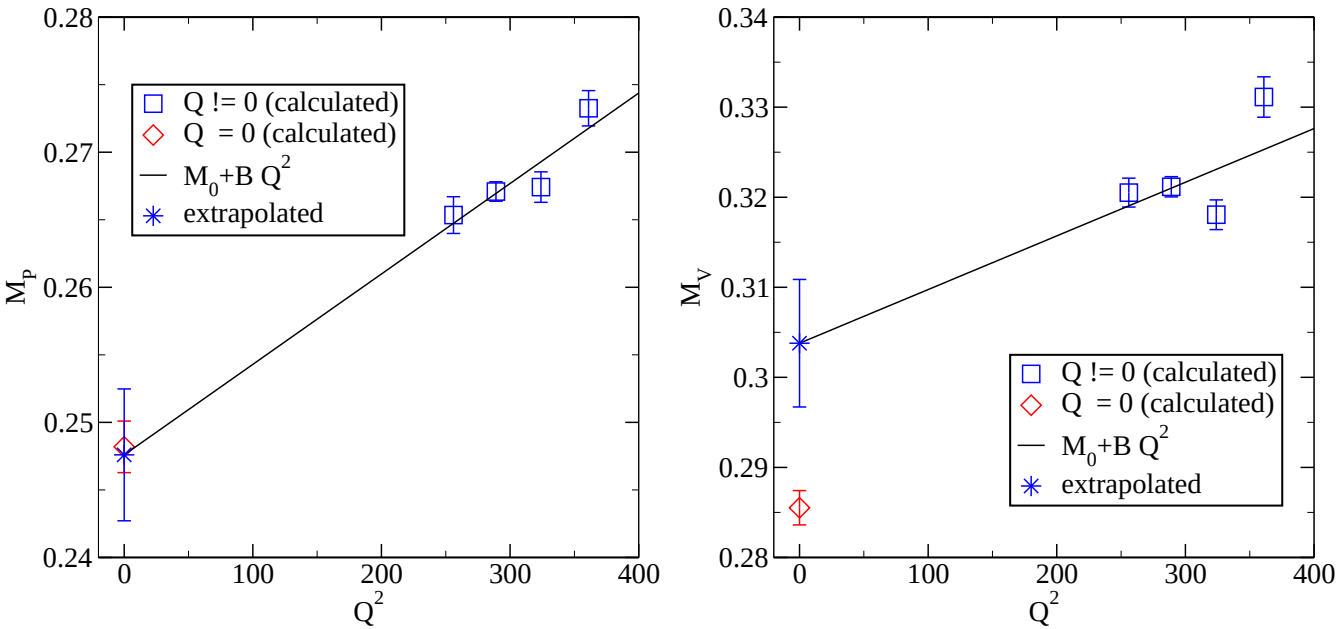
- The slight topological tunneling at $m_f = 0.025$ (D) allows us to analyze the data for different topological sectors separately.
- Given the limited number of measurements for each topology, we only block every trajectory, hence the errors will be underestimated.
- Nevertheless, such analysis will give us a crude idea how much the fixed topology may affect our results.

R. Brower, S. Chandrasekharan, John W. Negele, and U.J. Wiese. 2003 has the following formula for the Q-dependence

$$M_Q = M(0) + \frac{1}{2} M^{(2)}(0) \frac{1}{V\chi_t} \left(1 - \frac{Q^2}{V\chi_t}\right) + O\left(\frac{1}{V^3}\right).$$

χ_t is the topological susceptibility, $M^{(2)}(0)$ is the second derivative with respect to θ at $\theta = 0$, and $M(0)$ is the mass at $\theta = 0$ (or equivalently, at non-fixed Q).

Observed Q-Dependence



- The discrepancy between ordered and disordered ensembles can be described by the dependence on the fixed topology.
- To get results at $\theta = 0$, we still need to know the topological susceptibility $\rightarrow$ work in progress.
- Large discrepancies $\Rightarrow$ small χ_t? V too small?

Combining Datasets

Lacking the results for the topological susceptibility, can we still combine the two ensembles and obtain some best estimates of the "true" value, $M(0)$?

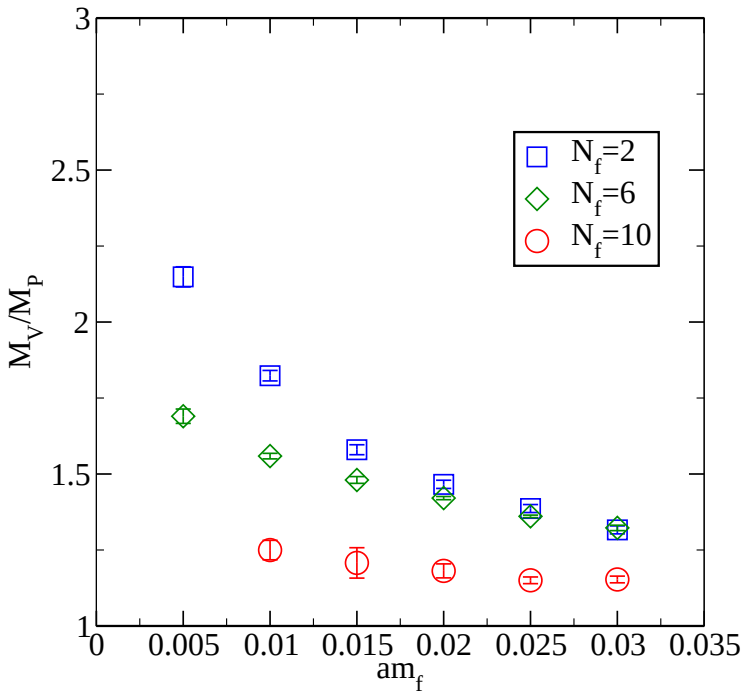
- From our fits, $M^{(2)}(0)$ is negative, so $M(0)$ is between the results for $Q = 0$ and $Q \neq 0$.
- Take $\overline{M} = (M_0 + M_Q)/2$ as the central value.
- Take the largest relative difference between M_Q and M_0 as systematic error.
- Combine statistical and systematic errors as final error.

Testing Conformality - Qualitatively

In a conformal theory with a small finite mass,

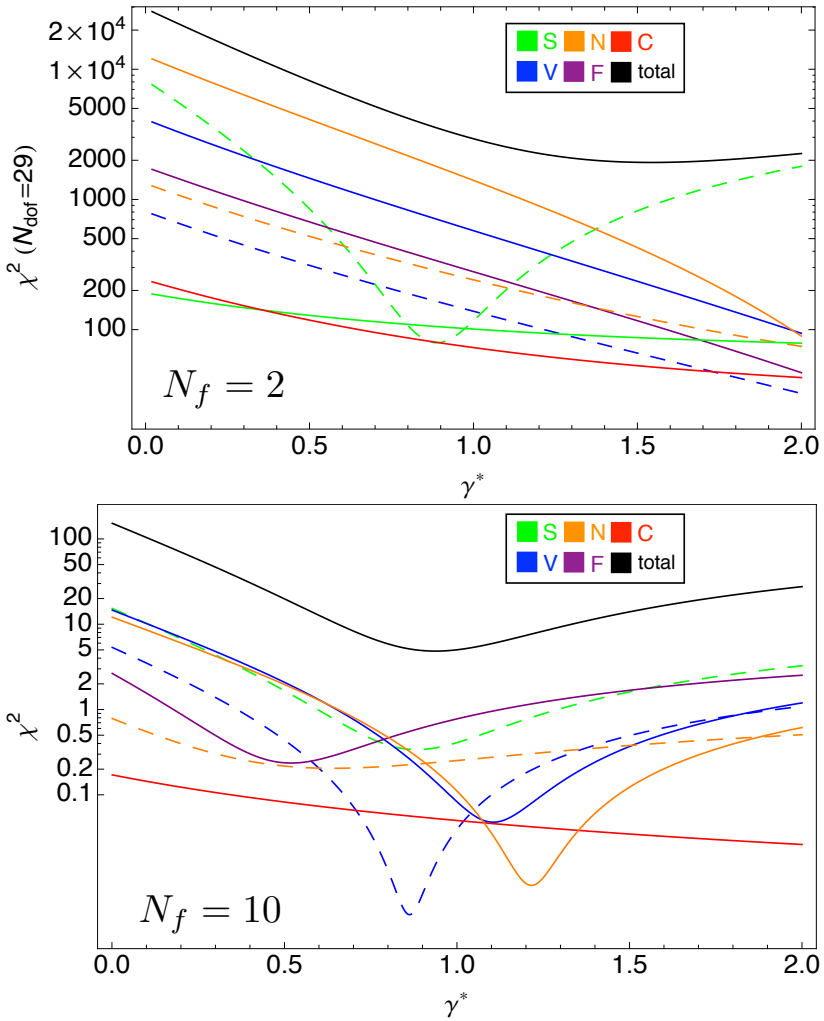
$$M_X = C_X m^{1/(1+\gamma^*)}$$

so the vector-to-pseudoscalar meson mass ratio M_V/M_P should look roughly constant.



- The $N_f = 2$ data are diverging as $m \to 0$, as expected since $M_P \to 0$.
- Similaryly for $N_f = 6$, though not as fast as $N_f = 2$.
- For $N_f = 10$ the ratios seem to be roughly constant.

Testing Conformality - Quantitatively



Obs.	$m_f \geq 0.010$	$m_f \geq 0.015$	$m_f \geq 0.020$
$\gamma^\star$	*1.45(15)*	0.94(16)	1.18(44)
[68% CI]	*[0.88,2.47]*	[0.68,1.44]	[1.01,1.69]
C_P	*0.978(9)*	1.44(21)	1.21(42)
C_V	*1.168(10)*	1.70(25)	1.42(49)
C_A	*1.429(13)*	2.14(32)	1.79(63)
C_N	*1.749(16)*	2.53(37)	2.10(73)
$C_{N^\star}$	*2.232(25)*	3.35(56)	2.87(1.02)
C_{FP}	*0.121(12)*	0.190(29)	0.164(57)
χ^2/d.o.f.	*69/31*	14/23	3/15

Finite volume effects may be large for $am_f \leq 0.01$.

Summary

- First SU(3) lattice simulations with $N_f = 6$ and 10 domain wall fermions have been performed.
- Hints of condensate enhancement and reduced S paramter have been seen with $N_f = 6$.
- With increasing N_f, the topology becomes more difficult to evolve. Effects of fixed topology are big for $N_f = 10$.
- Spectrum for $N_f = 10$ is consistent with conformality. But cannot rule out the chirally-broken scenario, as the masses are too heavy to make use of ChPT.

Future Work

- Calculate topological susceptibility at fixed Q for $N_f = 10$.
  S. Aoki *et al.*, 2007, 2008
- Quantative studies of finite volume effects
- Smaller quark masses